Roanoke Valley Governor’s School for Science and Technology
AP Calculus BC Competency List

(Last updated: June, 2022)

AP Calculus BC builds on the concepts learned in AP Calculus AB. The major themes include: advanced integration techniques, differential equations, series and approximation, parametric and polar functions presented numerically, geo-metrically, symbolically, and verbally as students learn to communicate the connections among these representations. Students are expected to obtain a qualifying score of 3, 4, or 5 on the AP Calculus BC exam at the end of this course. This course is taught using best practices in gifted education. Each competency is aligned with Hockett’s five principles of gifted education:

<u>Gifted Education Principles</u>:
(Hockett, J.A. (2009) “Curriculum for Highly Able Learners That Conforms to General Education and Gifted Education Quality Indicators.” *Journal of Education for the Gifted***.** Vol. 32, No. 3, p. 394-440)

1. High-quality curriculum for gifted learners uses a conceptual approach to organize or explore content that is discipline based and integrative.
2. High-quality curriculum for gifted learners pursues advanced levels of understanding beyond the general education curriculum through abstraction, depth, breadth, and complexity.
3. High-quality curriculum for gifted learners asks students to use processes and materials that approximate those of an expert, disciplinarian, or practicing professional.
4. High-quality curriculum for gifted learners emphasizes problems, products, and performances that are true to life, and outcomes that are transformational.
5. High-quality curriculum for gifted learners is flexible enough to accommodate self-directed learning fueled by student interests, adjustments for pacing, and variety.

External standards from AP Calculus BC Course Description were referenced when reviewing these competencies. To the right of each Enabling Objective is notation indicating alignment with external standards and a relative priority/proficiency rating from A (highest) to D (lowest).

COMPETENCY I – Limits and Continuity

Students will demonstrate an understanding of limits and continuity by evaluating functions for continuity and using analytic, graphing and numerical methods to calculate limits of functions.

Enabling Objectives:

Students will demonstrate an understanding of limits and continuity by evaluating functions for continuity and using analytic, graphing and numerical methods to calculate limits of functions.	1.1	A
Represent limits analytically using correct notation.	1.2	B
Interpret limits expressed in analytic notation.	1.2	B
Estimate limits of functions from graphs and tables.	1.3 1.4	C
Determine the limits of functions using limit theorems.	1.5	B
Determine the limits of functions using equivalent expressions for the function or the squeeze theorem.	1.6 1.8	C
Selecting an appropriate procedure for determining limits.	1.7	A
Apply all learning objectives relating to limits and translating mathematical information from a single representation or across multiple representations.	1.9	B
Justify conclusions about continuity at a point using the definition.	1.10 1.11	B
Determine intervals over which a function is continuous.	1.12	B
Determine values of x or solve for parameters that make discontinuous functions continuous, if possible.	1.13	B
Interpret the behavior of functions using limits involving infinity.	1.14 1.15	B
Explain the behavior of a function on an interval using the Intermediate Value Theorem.	1.16	A

COMPETENCY 2 – Fundamental Properties of Differentiation

Students will demonstrate an understanding of the fundamental properties of differentiation by explaining derivatives as rates of change, calculating derivatives using differentiation rules, and estimating derivatives of polynomials, trigonometric, exponential, and logarithmic functions.

Enabling Objectives:

Determine average rates of change using difference quotients.	2.1	B
Represent the derivative of a function as the limit of a difference quotient.	2.1 2.2	C
Determine the equation of a line tangent to a curve at a given point.	2.2	A
Approximate the zeros of a function using Newton's Method.	Exceeds standards	C
Estimate derivatives.	2.3	D
Explain the relationship between differentiability and continuity.	2.4	B
Calculate derivatives of familiar functions using the Power Rule, Constant Rule, and Sum and Difference Rule	2.5 2.6	A
Calculate derivatives of sin x, cos x, e^x , and ln x.	2.7	B
Calculate derivatives of products and quotients of differentiable functions.	2.8 2.9	A
Calculate derivatives of tan x, cot x, sec x, and csc x.	2.10	B

COMPETENCY 3 – Differentiation: Composite, Implicit, and Inverse Functions

Students will demonstrate an understanding of differentiation in composite, implicit, and inverse functions by differentiating composite functions, calculating derivatives of implicit and inverse functions, and calculating higher-order derivatives.

Enabling Objectives:

Calculate derivatives of compositions of differentiable functions using the Chain Rule.	3.1	A
Calculate derivatives of implicitly defined functions.	3.2	A
Calculate derivatives of inverse and inverse trigonometric functions.	3.3 3.4	B
Selecting an appropriate procedure for calculating derivatives.	3.5	B
Determine higher order derivatives of a function.	3.6	B

COMPETENCY 4 – Contextual Applications of Differentiation

Students will demonstrate an understanding of contextual applications of differentiation by interpreting derivatives in motion, analyzing rates and related rates of change, and approximating values of functions using local linearity and linearization.

Enabling Objectives:

Interpret the meaning of a derivative in context.	4.1	B
Calculate rates of change in applied contexts.	4.2 4.3	A
Calculate related rates in applied contexts.	4.4 4.5	A
Approximate a value on a curve using the equation of a tangent line.	4.6	C
Determine limits of functions that result in indeterminate forms by using L'Hopital's Rule.	4.7	A

COMPETENCY 5 – Applications of Differentiation

Students will demonstrate an understanding of applications of differentiation by calculating first and second derivatives, analyzing graph characteristics using the first and second derivatives, solving optimization problems, and analyzing implicit relations of functions.

Enabling Objectives:

Justify conclusions about functions by applying the Mean Value Theorem over an interval.	5.1	A
Justify conclusions about functions by applying the Extreme Value Theorem.	5.2	C
Determine intervals on which a function is increasing or decreasing by using the first derivative.	5.3	A
Use the First Derivative Test to determine local extrema.	5.4	A
Use the Candidates Test to determine absolute extrema.	5.5	B
Determine intervals on which a function is concave up or concave down by using the second derivative.	5.6	A
Use the Second Derivative Test to determine extrema.	5.7	B
Sketch graphs of functions and their derivatives.	5.8 5.9	B
Calculate optimization problems by finding the minimum and maximum values in applied contexts or analysis of functions.	5.10 5.11	A
Justify conclusions about the behavior of an implicitly defined function based on evidence from its derivatives	5.12	C

COMPETENCY 6 – Integration and Accumulation of Change

Students will demonstrate an understanding of integration and accumulation of change by explaining how the Fundamental Theorem connects differentiation and integration, calculating integrals using a variety of techniques, and evaluating improper integrals.

Enabling Objectives:

Interpret the meaning of areas associated with the graph of a rate of change in context.	6.1	A
Approximate a definite integral using geometric and numerical methods.	6.2	B
Interpret the limiting case of the Riemann sum as a definite integral.	6.3	A
Represent accumulation functions using definite integrals.	6.4 6.5	B
Calculate a definite integral using areas and properties of definite integrals.	6.6	A
Evaluate definite integrals analytically using the Fundamental Theorem of Calculus.	6.7	A
Determine antiderivatives of functions and indefinite integrals using knowledge of derivatives.	6.8	A
Determine antiderivatives of functions and indefinite integrals using substitution.	6.9	A
Determine antiderivatives of functions and indefinite integrals using long division and completing the square.	6.10	C
Determine antiderivatives of functions and indefinite integrals using integration by parts.	6.11	A
Determine antiderivatives of functions and indefinite integrals using partial fraction decomposition.	6.12	B
Evaluate an improper integral or determine that the integral diverges.	6.13	B
Selecting an appropriate procedure for antidifferentiation.	6.14	A

COMPETENCY 7 – Differential Equations

Students will demonstrate an understanding of differential equations by analyzing slope fields, solving differential equations using separation of variables, solving differential equations involving exponential and logistic growth and decay models, and estimating solutions to differential equations using Euler’s Method.

Enabling Objectives:

Interpret verbal statements of problems as differential equations involving a derivative expression.	7.1	C
Verify solutions to differential equations.	7.2	B
Estimate solutions to differential equations by using slope fields.	7.3 7.4	B
Estimate solutions to differential equations by using Euler’s Method.	7.5	A
Determine general and particular solutions to differential equations by using separation of variables.	7.6 7.7	A
Interpret the meaning of the exponential model in context.	7.8	C
Interpret the meaning of the logistic model in context.	7.9	C

COMPETENCY 8 – Applications of Integration

Students will demonstrate an understanding of integration applications by solving real-world applications and calculating applications involving area and volume.

Enabling Objectives:

Determine the average value of a function using definite integrals.	8.1	A
Determine values for positions and rates of change using definite integrals.	8.2	D
Interpret the meaning of a definite integral in accumulation problems.	8.3	B
Determine net change using definite integrals in applied contexts.	8.3	B
Calculate area between function and x-axis, area between two functions, and area between two functions that intersect in more than two points.	8.4 8.5 8.6	A
Calculate volumes of solids with known cross sections using definite integrals.	8.7 8.8	A
Calculate volumes of solids of revolution using the Disc and Washer methods.	8.9 8.10 8.11 8.12	A
Calculate volumes of solids of revolution using the Shell method.	Exceeds standards	C
Determine the length of a curve in the plane defined by a function using a definite integral.	8.13	B

COMPETENCY 9 – Parametric, Polar, and Vector-Valued Equations

Students will demonstrate an understanding of parametric, polar, and vector-valued equations by defining parametric and vector-valued functions to define planar motion, solving motion problems, and calculating area between two polar curves.

Enabling Objectives:

Calculate first and second derivatives of parametric functions.	9.1 9.2	A
Determine the length of a curve in the plane defined by parametric functions using a definite integral.	9.3	C
Calculate derivatives of vector-valued functions.	9.4	B
Determine a particular solution given a rate vector and initial conditions.	9.5	B
Determine values for positions and rates of change in problems involving planar motion.	9.6	B
Calculate derivatives of functions written in polar coordinates.	9.7	A
Calculate areas of regions defined by polar curves using definite integrals.	9.8 9.9	A

COMPETENCY 10 – Infinite Sequences and Series

Students will demonstrate an understanding of infinite sequences and series by evaluating the convergence and divergence of a series, calculating and estimating the sum of a series, and constructing and using Taylor Polynomials.

Enabling Objectives:

Determine whether a series converges or diverges by examining a sequence.	10.1	B
Determine whether a series converges or diverges using Geometric Series Test.	10.2	A
Determine whether a series converges or diverges using n^{th} Term Test.	10.3	A
Determine whether a series converges or diverges using the Integral Test.	10.4	A
Determine whether a series converges or diverges using the p-Series Test.	10.5	A
Determine whether a series converges or diverges using the Direct Comparison Test and the Limit Comparison Test.	10.6	A
Determine whether a series converges or diverges the Alternating Series Test.	10.7	A
Determine whether a series converges or diverges using the Ratio Test.	10.8	A
Determine whether a series converges or diverges the Root Test.	Exceeds Standards	C
Determine absolute or conditional convergence of an alternating series.	10.9	A
Examine the error bound on an alternating series.	10.10	B
Represent a function at a point as a Taylor polynomial.	10.11	A
Approximate function values using a Taylor polynomial.	10.11	B
Determine the error bound associated with a Taylor polynomial approximation by using Lagrange error bound.	10.12	B
Determine the radius of convergence and interval of convergence for a power series.	10.13	A
Represent a function as a Taylor series or a Maclaurin series.	10.14	A
Represent a given function as a power series	10.15	B