Roanoke Valley Governor's School for Science and Technology
AP Calculus BC
Competency List, 2018-2019

(Last updated: 8-5-2014)

AP Calculus BC builds on the concepts learned in AP Calculus AB. The major themes include: advanced integration techniques, differential equations, series and approximation, parametric and polar functions presented numerically, geo-metrically, symbolically, and verbally as students learn to communicate the connections among these representations. Students are expected to obtain a qualifying score of 3, 4, or 5 on the AP Calculus BC exam at the end of this course. This course is taught using best practices in gifted education. Each competency is aligned with Hockett's five principles of gifted education:

Gifted Education Principles:
(Hockett, J.A. (2009) "Curriculum for Highly Able Learners That Conforms to General Education and Gifted Education Quality Indicators." *Journal of Education for the Gifted***.** Vol. 32, No. 3, p. 394-440)

1. High-quality curriculum for gifted learners uses a conceptual approach to organize or explore content that is discipline based and integrative.
2. High-quality curriculum for gifted learners pursues advanced levels of understanding beyond the general education curriculum through abstraction, depth, breadth, and complexity.
3. High-quality curriculum for gifted learners asks students to use processes and materials that approximate those of an expert, disciplinarian, or practicing professional.
4. High-quality curriculum for gifted learners emphasizes problems, products, and performances that are true to life, and outcomes that are transformational.
5. High-quality curriculum for gifted learners is flexible enough to accommodate self-directed learning fueled by student interests, adjustments for pacing, and variety.

COMPETENCY I
Use the concept of local linearity to generalize the concept of a linear slope to all functions.
Enabling Objectives:

1. Use the calculator to graph a non-linear function.
2. Approximate the value of a function at a particular point.
3. Identify whether or not a function is locally linear at a point.
4. Apply the linear concept of slope , $\Delta y / \Delta x$, to estimate the value of the slope of a point on a nonlinear function.
5. Relate the concept of limit to the slope of a line.
6. Find the value of the slope using the difference quotient.

COMPETENCY II

Define and apply the properties of limits of functions.

Enabling Objectives:

1. Calculate the limit of both positive and negative constants.
2. Calculate the limit of a sum of functions.
3. Calculate the limit of a product of functions.
4. Calculate the limit of a quotient of functions.
5. Calculate limits at infinity.
6. Calculate infinite limits.
7. Determine if a limit is undefined.
8. Apply the Squeeze Theorem.

COMPETENCY III

Define and apply the properties of continuity of a function.

Enabling Objectives:

1. Apply the definition of continuity to determine continuity at a point
2. Apply the definition of continuity to determine continuity over a closed interval
3. Use definition of continuity to prove the Mean Value Theorem
4. Use graphs to determine continuity and discontinuity

COMPETENCY IV

Identify the connection between the slope of a function and the derivative.

Enabling Objectives:

1. State all forms of the definition of the derivative.
2. Identify the relationship between differentiability and continuity.
3. Relate a function being differentiable at a point to a differentiable function.
4. Identify differentiable points on a function using the concept of local linearity.

COMPETENCY V

Apply techniques of differentiation to calculate the derivative of all types of functions.

Enabling Objectives:

1. Use the limit definition to calculate the derivative.
2. Use the power rule to calculate derivatives.
3. Use the power rule to calculate the derivatives of sums and differences.
4. Prove the product rule using limits.
5. Prove the quotient rule using limits.
6. Use the product rule to calculate derivatives.
7. Use the quotient rule to calculate derivatives.
8. Calculate the derivative of the inverse of functions
9. Use the techniques of differentiation to solve a variety of initial value problems.
10. Use the techniques of differentiation to calculate the derivatives of implicitly defined functions.
11. Use the techniques of differentiation to calculate the derivatives of trigonometric and inverse trigonometric functions.
12. Use the techniques of differentiation to calculate the derivatives of exponential functions.
13. Use the techniques of differentiation to calculate the derivatives of logarithmic functions.

COMPETENCY VI

Sketch the graph of a function by analyzing the extreme values of a function using the properties of the derivative.

Enabling Objectives:

1. Define and determine critical point(s).
2. Identify extrema as a subset of critical points.
3. Identify extrema on a graph as global maximums and minimums.
4. Identify extrema on a graph as local maximums and minimums.
5. Verify the existence of extrema using the closed interval, continuity definition.
6. Use the first derivative to determine critical points of a function.
7. Use the first derivative to test critical points.
8. Use the second derivative to test critical points.

COMPETENCY VII

Apply the concept of rate as a function of time both analytically and geometrically to other rates of change.

Enabling Objectives:

1. Use a table to demonstrate non-constant rates of change, negative rates of change, and estimated rates of change.
2. Generalize the distance vs. time relationship demonstrated by a falling object.
3. Identify the connection between a distance vs. time graph and velocity.
4. Solve rate of change problems.

COMPETENCY VIII

Identify the derivative as a function that describes properties of an original equation graphically.

Enabling Objectives:

1. Use the graph of the derivative to identify points on the original function where the slope is zero.
2. Use the graph of the derivative to identify points on the original function where the slope is positive.
3. Use the graph of the derivative to identify points on the original function where the slope is negative.
4. Determine whether a function is increasing, decreasing or neither from the graph of the first derivative.
5. Determine the steepness of a function from the magnitude of the value of the derivative.

COMPETENCY IX

Identify the derivative as a function that describes properties of an original equation algebraically.

Enabling Objectives:

1. Use basic algebraic derivative formulas to calculate an equation for the derivative.
2. Identify individual pieces of compound functions.
3. Apply the chain rule to find the derivative of composite functions.

4. Use the equation of the derivative to identify points on the original function where the slope is zero.
5. Use the equation of the derivative to identify points on the original function where the slope is positive.
6. Use the equation of the derivative to identify points on the original function where the slope is negative.

COMPETENCY X

Find optimal values for area, shape, cost and other real-world applications using the derivative.

Enabling Objectives:

1. Graph an equation for a physical property and estimate the minimum or maximum value.
2. Use an equation with a fixed relationship between variables to write the equation to be optimized in terms of one variable.
3. Take the derivative of the optimal equation to determine extreme values.
4. Use a derivative test to verify the extreme value as a maximum or a minimum.
5. Use the formula for the derivative to calculate the slope of tangent and normal lines to a function.
6. Analyze distance, velocity and acceleration problems through the use of the derivative and antiderivative
7. Solve related rate problems.
8. Solve applied optimization problems.

COMPETENCY XI

Apply derivatives in order to solve mathematical and scientific problems.

Enabling Objectives:

1. Use linearization to approximate a function for a given value of x.
2. Calculate Δy for a given change in x.
3. Use differentials to approximate Δy.
4. Compare the value of the differential, dy, to the actual change in y.
5. Find the differential of a function using differentiation formulas.

COMPETENCY XII

Find and verify solutions to initial value problems and apply to the real world.

Enabling Objectives:

1. Verify that a given equation solves an initial value problem.
2. Identify exponential functions as solutions to differential equations.
3. Identify the natural exponential function as a solution to the differential equation y'=ky.
4. Use the inverse function relationship between the natural logarithmic function and the natural exponential function to analyze initial value application problems.
5. Solve separable differential equations.

COMPETENCY XIII

Evaluate estimating and calculating techniques for Integration.

Enabling Objectives:

1. Use the idea of infinite approximations to find the value of a definite integral.

2. Use u-substitution(s) to evaluate definite and indefinite integrals.
3. Evaluate definite and indefinite integrals of algebraic, trigonometric, exponential and inverse functions using integration formulas.
4. Apply the Fundamental Theorem of calculus to real-world applications.
5. Determine the average (mean) value of a function on an interval.

COMPETENCY XIV

Apply the integral as a technique for solving real-world problems.

Enabling Objectives:

1. Calculate the area under a graph.
2. Calculate the area between two functions.
3. Calculate the area between several functions.
4. Revolve the region between functions about a line and use integrals to calculate the volume of the resulting solid.
5. Calculate volumes of solids of revolution using the following methods: plane slicing and cross-sections.
6. Investigate the Mean Value Theorem for definite integrals.
7. Use Riemann Sums and Trapezoid rule to numerically evaluate integrals, both manually and using appropriate technology.

COMPETENCY XV

Explore advanced integration techniques for evaluating definite and indefinite integrals.

Enabling Objectives:

1. Use integration by parts to evaluate definite and indefinite integrals.
2. Integrate rational functions using linear and quadratic factors to find a partial fraction decomposition.
3. Complete the square to integrate quadratic functions.
4. Identify improper integrals with infinite discontinuities and infinite limits of integration.
5. Define convergence and divergence of improper integrals.
6. Evaluate improper integrals.
7. Calculate the arc length of rectangular functions.
8. Recognize limits that produce indeterminant forms.
9. Apply L'Hopital's rule to evaluate limits.

COMPETENCY XVI

Use the properties of sequences and series to determine their convergence or divergence and to calculate the sums of series.

Enabling Objectives:

1. Define monotonic and bounded sequences.
2. Apply the Squeeze Theorem and the Absolute Value Theorem for sequences.
3. Use limits to determine the convergence or divergence of sequences.
4. Identify the properties of infinite series.
5. Use sequences of partial sums to determine the sums of infinite series.
6. Identify the following types of series and their properties:
    a. Infinite series
    b. Geometric series, including formula for the nth partial sum

    c. Alternating series
    d. Telescoping series
    e. Harmonic and p-series
7. Determine the convergence or divergence of series using:
    a. Absolute convergence
    b. nth-term test
    c. Geometric series test
    d. Telescoping series test
    e. p-series test
    f. Alternating series test
    g. Integral test
    h. Root test
    i. Ratio test
    j. Direct comparison test
    k. Limit comparison test

COMPETENCY XVII

Use Taylor and MacLaurin polynomials and power series to approximate functions.

Enabling Objectives:

1. Identify Taylor and MacLaurin polynomials and the general formula for each.
2. Find Taylor and MacLaurin polynomials of degree n and centered at c to approximate functions.
3. Use appropriate graphing software to explore Taylor and MacLaurin approximations.
4. Use Taylor and MacLaurin polynomials to approximate function values at a point.
5. Calculate the error of Taylor and MacLaurin approximations.
6. Identify a power series and determine its convergence.
7. Determine the radii and intervals of convergence for power series.
8. Represent functions as sums of power series.
9. Perform differentiation and integration on power series.
10. Express Taylor and MacLaurin polynomials as series.
11. Error analysis for series.

COMPETENCY XVIII

Explore the basic foundations of differential equations in order to solve them.

Enabling Objectives:

1. Use separation of variables to derive the exponential formula for growth and decay.
2. Explore appropriate applications of exponential growth, including:
    a. Newton's Law of Cooling
    b. Population growth
    c. Radioactive decay
3. Use separation of variables to find general solutions for differential equations.
4. Use initial values to find specific solutions to differential equations.
5. Verify solutions to differential equations.
6. Examine direction fields to explore graphically the solutions to differential equations, using technology as appropriate.
7. Solve logistic differential equations and use them in modeling.

COMPETENCY XIX

Define a function in parametric form.

Enabling Objectives:

1. Draw graphs manually to explore the relationship between rectangular and parametric coordinates.
2. Sketch a plane curve and label its orientation from its parametric equations.
3. Explore the effect of the parameter on the graph by applying situations involving time, angles and other parameters.
4. Find parametric equations to describe a graph.
5. Convert equations between rectangular and parametric coordinates.
6. Identify smooth and piecewise smooth curves.
7. Find the parametric form of the derivative for a plane curve.
8. Identify the points of horizontal and vertical tangency using derivatives.
9. Find the arc length of a plane curve.

COMPETENCY XX

Study the polar coordinate system in the context of calculus.

Enabling Objectives:

1. Identify and describe the polar coordinate system.
2. Plot a point given its polar coordinates.
3. Find the four sets of polar coordinates describing a point.
4. Convert coordinates between rectangular and polar and vice versa.
5. Convert equations between rectangular and polar and vice versa.
6. Graph equations in polar coordinates.
7. Know the rules for symmetry of equations.
8. Find the relative extrema of r.
9. Find the slope of a polar graph.
10. Identify horizontal and vertical tangents to polar curves.
11. Find the tangents at the pole.
12. Identify and sketch the basic types of polar curves from their equations.
13. Use integration to calculate area of regions bounded by polar curves.

COMPETENCY XXI

Solve application problems using vectors and vector-valued functions.

Enabling Objectives:

1. Find the velocity vector given the position vector.
2. Find the position vector given the velocity vector and initial position.
3. Determine when movement of a particle is horizontal or vertical.
4. Find the acceleration vector.
5. Find the speed of a particle by finding the length of the velocity vector.
6. Find the slope of the tangent line to a vector at a given point.
7. Find the derivative of a vector-valued function at a point.
8. Integrate vector-valued functions.
9. Use vector-valued functions to describe velocity and acceleration.

10. Solve problems involving the position function for a projectile.

POST AP EXAM TOPICS:

1. Use identities and reduction formulas to integrate higher order trigonometric functions.
2. Use trigonometric substitutions to evaluate integrands involving square roots of quadratic functions.
3. Expand calculus applications through the use of calculators and computers by: entering a program into a TI-84 graphing calculator, explaining the purpose of each line in a calculator program, adapting a program that satisfies the need of a singular situation to be useful in a general situation, using the TI-Link and inter-calculator link, identifying when to write programs for the calculator to simplify learning, writing programs for the calculator which apply calculus concepts, and using computer software to assist in the application of calculus concepts. Students already competent with programming with the TI-84 may elect to manipulate a TI-89.
4. Use Newton's Method to find the zeros of a function, manually and using appropriate technology.
5. Calculate the arc length and area of a surface of revolution for polar curves.
6. Use a table appropriately to evaluate integrals.
7. Use Simpson's rule to numerically evaluate integrals, both manually and using appropriate technology.
8. Find partial derivatives of functions.
9. Define homogeneous differential equations.
10. Use $y = vx$ substitution and separation of variables to solve differential equations.
11. Find orthogonal trajectories for a family of curves.
12. Calculate the area of a surface of revolution generated from a plane curve.