

Roanoke Valley Governor's School for Science and Technology

RVGS Pre-calculus

Competency List

(Last updated: 08/2018)

RVGS Pre-calculus is designed to prepare students for AP Calculus. The major themes include functional analysis (polynomial and rational functions, exponential functions, logarithmic functions, trigonometric functions, polar functions and parametric functions), conic sections, systems of equations and inequalities, matrices, sequences and series, and mathematical modeling.

This course is taught using best practices in gifted education. Each competency is aligned with Hockett's five principles of gifted education:

Gifted Education Principles:

(Hockett, J.A. (2009) "Curriculum for Highly Able Learners That Conforms to General Education and Gifted Education Quality Indicators." *Journal of Education for the Gifted*. Vol. 32, No. 3, p. 394-440)

1. High-quality curriculum for gifted learners uses a conceptual approach to organize or explore content that is discipline based and integrative.
2. High-quality curriculum for gifted learners pursues advanced levels of understanding beyond the general education curriculum through abstraction, depth, breadth, and complexity.
3. High-quality curriculum for gifted learners asks students to use processes and materials that approximate those of an expert, disciplinarian, or practicing professional.
4. High-quality curriculum for gifted learners emphasizes problems, products, and performances that are true to life, and outcomes that are transformational.
5. High-quality curriculum for gifted learners is flexible enough to accommodate self-directed learning fueled by student interests, adjustments for pacing, and variety.

Unit 1: Exponential and logarithmic equations and functions

Competencies:

1. Simplify exponential and logarithmic expressions.
2. Solve exponential and logarithmic equations.
3. Graph exponential and logarithmic functions.
4. Apply base e logarithms to real-world situations.
5. Use exponential functions of base e in real-world applications.
6. Develop and apply the Change of Base formula.
7. Derive and apply properties of logarithms
8. Given a real-world situation relating two variables, use an appropriate function as a mathematical model.
9. Demonstrate the application of the concepts in Calculus.

Unit 2: Trigonometric functions

Competencies:

1. Develop and apply the concept of a wrapping function that pairs arc length with a point on the unit circle.
2. Define radian.
3. Develop and apply the relationships among central angle, radius, and arc length in a circle.
4. Define the six circular trigonometric functions.

- Determine the exact values of the six circular trigonometric functions of quadrantal angles and angles in the $\frac{\pi}{6}$, $\frac{\pi}{4}$ and $\frac{\pi}{3}$ families.
- Determine the value of the six trigonometric functions for a given angle.
- Apply inverse trigonometric functions to determine the angle measure or arc length of given values.
- Graph the six parent trigonometric functions and their inverses.
- Graph transformed trigonometric functions by applying the concepts of period, amplitude, phase shift and reflections.
- Determine the trigonometric function when given its graph.
- Investigate real world applications of sinusoidal functions.
- Represent the relationship between trigonometric functions and their inverses.
- Simplify expressions containing trigonometric functions and inverses.
- Demonstrate the application of the concepts in Calculus.

Unit 3: Trigonometric identities and equations

Competencies:

- Know and apply the Quotient, Reciprocal, Pythagorean, Even-Odd, Sum and Difference, Double, and Half-Angle formulas to write equivalent expressions, to prove identities, and to solve equations.
- Solve trigonometric equations over a given domain algebraically by applying appropriate identities and verify graphically.
- Demonstrate the application of the concepts in Calculus.

Unit 4: Trigonometry properties of triangles

Competencies:

- Solve right triangles.
- Solve oblique triangles using Law of Sines and Law of Cosines.
- Determine the area of an oblique triangle.
- Apply concepts of trigonometric properties of triangles to real life situations.
- Demonstrate the application of the concepts in Calculus.

Unit 5: Systems of equations and inequalities

Competencies:

- Solve systems of nonlinear equations and inequalities graphically and algebraically.
- Solve systems of linear equations by performing row operations on augmented matrices.
- Calculate the determinant of a matrix.
- Calculate the inverse of a matrix.
- Solve a system of linear equations using Cramer's Rule and the inverse matrix method.
- Determine the partial fraction decomposition of a rational expression.
- Create and solve the system of inequalities to optimize an objective function representing a real world application.
- Demonstrate the application of the concepts in Calculus.

Unit 6: Conic sections: circles, parabolas, ellipses, and hyperbolas

Competencies:

- Define each conic section as a locus of points.
- Use the equation to graph conic sections.
- Use the graph or information from the graph to determine the equation of the conic section.
- Analyze and understand the eccentricity of each conic section.
- Given the general form, identify the conic section.

6. Convert from the general form of a conic section to its standard form.
7. Apply concepts of conic sections to real life situations.
8. Demonstrate the application of the concepts in Calculus.

Unit 7: Polar and parametric functions

Competencies:

1. Plot points on the polar coordinate system.
2. Convert between polar coordinates and rectangular coordinates.
3. Convert equations between polar form and rectangular form.
4. Identify and graph specific polar equations: line, circle, cardioid, limaçon, and lemniscate.
5. Graph polar equations.
6. Determine the point of intersection given 2 polar equations.
7. Graph parametric equations.
8. Convert equations between parametric form and rectangular form.
9. Demonstrate the application of the concepts in Calculus.

Unit 8: Sequences and series

Competencies:

1. Given an arithmetic or geometric series, determine S_n , the n^{th} partial sum.
2. Given a geometric series, tell whether it converges. If it does converge, determine the limit to which it converges.
3. Given a repeating decimal, write it as a convergent geometric series, and find a rational number equal to the decimal.
4. Interpret sequences and series graphically.
5. Demonstrate the application of the concepts in Calculus.

Unit 9: Polynomial and rational functions, continuity and limits

Competencies:

1. For a polynomial function, determine the zeros and their multiplicity, end behaviors, x -intercepts, y -intercept, maximum number of turns algebraically and graphically.
2. For a rational function determine the zeros, y -intercept, x -intercepts, asymptotes, maximum and minimum values, intervals for which the function is increasing or decreasing, and intervals for which the function is positive or negative algebraically and graphically.
3. Investigate the concept of continuity and concavity, graphically and algebraically.
4. Investigate the concept of limits (left-hand, right-hand, and infinite), graphically and algebraically, using rational functions.
5. Calculate the limit of an algebraic function, if it exists, as the variable approaches either a finite number or infinity.
6. Demonstrate the application of the concepts in Calculus.